COMPLETELY (q,p)-MIXING MAPS

JAVIER ALEJANDRO CHÁVEZ-DOMÍNGUEZ

Abstract. Several important results for p-summing operators, such as Pietsch's composition formula and Grothendieck's theorem, share the following form: there is an operator T such that $S \circ T$ is p-summing whenever S is q-summing. Such operators were called (q,p)-mixing by Pietsch, who studied them systematically. In the operator space setting, G. Pisier's completely p-summing maps correspond to the p-summing operators between Banach spaces. A natural modification of the definition yields the notion of completely (q,p)-mixing maps, already introduced by K.L. Yew, which is the subject of this paper. Some basic properties of these maps are proved, as well as a couple of characterizations. A generalization of Yew's operator space version of the Extrapolation Theorem is obtained, via an interpolation-style theorem relating different completely (q,p)-mixing norms. Finally, some composition theorems for completely p-summing maps are proved.

1. Introduction

The theory of (absolutely) p-summing operators traces its roots back to the works of A. Grothendieck in the 50's [Gro53], but it was not until the late 60's that A. Pietsch clearly defined such operators and started the study of their fundamental properties [Pie67]. Throughout the years, p-summing operators have rightfully earned their place as one of the cornerstones of modern Banach space theory. They have given rise to a wide array of results not only within the realm of Banach spaces, but also reaching into other areas like harmonic analysis, probability and operator theory. An excellent reference for the general theory of p-summing operators is the book [DJT95]. One of the fundamental properties of p-summing operators, already present in Pietsch's seminal paper [Pie67], is the composition theorem: whenever $p,q,r \in [1,\infty]$ satisfy $1/p = 1/q + 1/r$, the composition of a q-summing operator followed by an r-summing operator is p-summing. Another celebrated theorem, due to A. Grothendieck, states that every continuous linear operator from L_1 into a Hilbert space is 1-summing; therefore, any 2-summing operator with an L_1

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space as domain is 1-summing. More generally, by a theorem of B. Maurey, any 2-summing operator defined on a cotype 2 space is 1-summing. These results share a common theme: in each case we have an operator T (often the identity on a space) and certain values of p and q with the property that $S \circ T$ is p-summing whenever S is q-summing.

Inspired by ideas of Maurey [Mau74], Pietsch [Pie80, Chap. 20] systematically studied the situation described in the previous paragraph and called such operators (q, p)-mixing. Another exposition of the subject, with a more "tensorial" point of view, can be found in [DF93, Sec. 32]. An analogous study was done by the author in the setting of Lipschitz p-summing and Lipschitz (q, p)-mixing operators [CD].

In the theory of operator spaces, p-summing operators are replaced by the completely p-summing maps of Pisier [Pis98]. There is, of course, a natural notion of completely (q, p)-mixing maps that has already been introduced in [Yew08], but no systematic study of these maps was done there. The present paper aims to fill that void, and it is structured as follows. We start by recalling some basic notation and results from operator space theory, before formally introducing the definition of completely p-summing maps and proving some of their elementary properties. Afterwards, two different characterizations of completely (q, p)-mixing maps are presented. The first one is a "domination" result along the lines of the Pietsch domination theorem for completely p-summing maps due to Pisier [Pis98]. The second one does not clearly correspond to any of the characterizations in the classical case that can be found in [DF93, Sec. 32], but nevertheless it is used to prove an "interpolation" theorem relating different completely (q, p)-mixing norms which actually is inspired by the classical case. As a byproduct, a strengthening of Yew's quantized extrapolation theorem [Yew08, Thm. 8] is obtained. In the final section, several composition theorems are proved, culminating with a composition theorem for completely p-summing maps: if $1/r = 1/p + 1/q$, then the composition of a completely p-summing map and a completely q-summing one is completely r-summing.

2. Notation and preliminaries

We only assume familiarity with the basic theory of operator spaces; Pisier's book [Pis03] is an excellent reference for that. We will follow very closely the Pisier's notation from [Pis98, Pis03]. The letters E, F and G will always denote operator spaces. For an operator space E, a Hilbert space K and $1 \leq p \leq \infty$, let us define the spaces S_p, $S_p[E]$ and $S_p(K)$. For $1 < p < \infty$, S_p (resp. $S_p(K)$) denotes the space of Schatten class operators in ℓ_2 (resp. on K). In the case $p = \infty$, we denote by S_∞ (resp. $S_p(K)$) the space of all compact operators on ℓ_2 (resp. on K) with the operator space structure inherited from $B(\ell_2)$ (resp. $B(K)$). We define $S_\infty[E]$ as the minimal operator space tensor product of S_∞ and E, and $S_1[E]$ as the operator space projective tensor product of S_1 and E. In the case $1 < p < \infty$, $S_p[E]$ is defined via complex interpolation between $S_\infty[E]$ and $S_1[E]$.

Let E, F be operator spaces and $u : E \to F$ a linear map. For $1 \le p \le \infty$, we will say that u is *completely p-summing* if the mapping

$$I_{S_p} \otimes u : S_p \otimes_{\min} E \to S_p[F]$$

is bounded, and we denote its norm by $\pi_p^o(u)$. By a result of Pisier [Pis98, Corollary 5.5], in the case $1 \le p < \infty$ we in fact have that the cb-norm and the norm of the map $I_{S_p} \otimes u$ are equal. For notational convenience, we will use the convention $\pi_\infty^o(\cdot) = \|\cdot\|_{\mathrm{cb}}$. Completely p-summing maps satisfy the ideal property (that is, $\pi_p^o(uvw) \le \|u\|_{\mathrm{cb}}\, \pi_p^o(v)\, \|w\|_{\mathrm{cb}}$ whenever the composition makes sense), and being completely p-summing is a local property: the completely p-summing norm of $u : E \to F$ is equal to the supremum of the completely p-summing norms of the restrictions of u to finite-dimensional operator subspaces of E. In fact,

$$\pi_p^o(u : E \to F) = \sup\left\{\pi_p^o(uT) \ : \ T : S_{p'}^n \to E, n \ge 1, \|T\|_{\mathrm{cb}} \le 1\right\}.$$

The following theorem, due to Pisier [Pis98, Thm. 5.1] is an important characterization of completely p-summing maps.

Theorem 2.1 (Pietsch domination)**.** *Assume $E \subseteq B(H)$. Let $u : E \to F$ be a completely p-summing map ($1 \le p < \infty$) and let $C = \pi_p^o(u)$. Then there is an ultrafilter $\mathcal{U}$ over an index set I and families $(a_\alpha)_{\alpha\in I}$, $(b_\alpha)_{\alpha\in I}$ in the unit ball of $S_{2p}(H)$ such that for all $n \in \mathbb{N}$ and all (x_{ij}) in $M_n(E)$ we have*

$$\left\|\left[(ux_{ij})\right]\right\|_{S_p^n[F]} \le C \lim_{\mathcal{U}} \left\|\left[(a_\alpha x_{ij} b_\alpha)\right]\right\|_{S_p(\ell_2^n\otimes H)} \tag{2.1}$$

and

$$\left\|\left[(ux_{ij})\right]\right\|_{M_n[F]} \le C \lim_{\mathcal{U}} \left\|\left[(a_\alpha x_{ij} b_\alpha)\right]\right\|_{M_n(S_p(H))}. \tag{2.2}$$

Conversely, if an operator u satisfies either (2.1) *or* (2.2) *then it is completely p-summing with $\pi_p^o(u) \le C$.*

One consequence of the domination theorem is the monotonicity of p-summing norms: if $1 \le p \le q$ and u is completely p-summing, then u is completely q-summing and moreover $\pi_q^o(u) \le \pi_p^o(u)$. The standard (although not canonical) example of a completely p-summing map is a multiplication map. To be precise, we have [Pis98, Prop. 5.6]

Theorem 2.2. *Let K be any Hilbert space. Consider a, b in $S_{2p}(K)$ and let $M(a,b) : B(K) \to S_p(K)$ be the operator defined by $M(a,b)x = axb$ for all x in $B(K)$. Then $\pi_p^o\big(M(a,b)\big) \le \|a\|_{S_{2p}(K)}\, \|b\|_{S_{2p}(K)}$.*

Following [Jun96], we say that a linear map $u : E \to F$ is *completely p-nuclear* (denoted $u \in \mathcal{N}_p^o(E,F)$) if there exists a factorization of u as

$$E \xrightarrow{\alpha} S_\infty \xrightarrow{M(a,b)} S_p \xrightarrow{\beta} F$$

with $a, b \in S_{2p}$ and α, β completely bounded maps. The completely p-nuclear norm of u is defined as

$$\nu_p^o(u) = \inf \left\{ \|\alpha\|_{\mathrm{cb}} \|a\|_{S_{2p}} \|b\|_{S_{2p}} \|\beta\|_{\mathrm{cb}} \right\}$$

where the infimum is taken over all factorizations of u as above.

3. Definition and elementary properties

Let $1 \le p, q \le \infty$. A map $u : E \to F$ is said to be *completely (q,p)-mixing with constant K* if for any operator space G and any completely q-summing map $v : F \to G$, the composition $v \circ u$ is a completely p-summing map and $\pi_p^o(v \circ u) \le K \pi_q^o(v)$. The *completely (q,p)-mixing norm* of u is the smallest such K and will be denoted by $\mathfrak{m}_{q,p}^o(u)$. Note that it is indeed a norm.

This definition (albeit worded in a different way) appears in [Yew08], where several upper and lower bounds for the completely $(2,p)$-mixing norms of the identity on OH_n are computed (for $1 < p < 2$). For an infinite-dimensional example of a completely mixing map, Junge and Parcet prove in [JP10, Corollary A2] that the identity map on the operator Hilbert space OH is completely $(q,1)$-mixing for any $1 < q < 2$ (in sharp contrast with the commutative case, Yew [Yew08] proved that this same map is not completely $(2,1)$-mixing). In fact Junge and Parcet proved a more general result, and in order to state it we will need some definitions. A map $u : E \to F$ is called *completely $(q,1)$-summing* if

$$\pi_{q,1}^{\mathrm{cb}}(u) := \|id \otimes u : \ell_1 \otimes_{\min} E \to \ell_q(F)\|_{\mathrm{cb}} < \infty,$$

and it is said to have cb-*cotype q* if

$$c_q^{\mathrm{cb}}(u) = \|\iota \otimes u : \mathrm{Rad}_q(E) \to \ell_q(F)\|_{\mathrm{cb}} < \infty,$$

with

$$\mathrm{Rad}_q(E) = \Big\{ \sum_j \varepsilon_j x_j \ : \ x_j \in E \Big\} \subset L_q(E)$$

where the ε_j's are independent ± 1 Bernoulli random variables, and $\iota(\varepsilon_j) = \delta_j$ where the δ_j's form the canonical basis of ℓ_q. If a map u has cb-cotype q then it is completely $(q,1)$-summing, and moreover $\pi_{q,1}^{\mathrm{cb}}(u) \le c_q^{\mathrm{cb}}(u)$ [JP10, Lemma 3.1]. The following result is a straightforward generalization of [JP10, Cor. 3.7].

Theorem 3.1. *Let $p \geq 2$. If $u : E \to F$ is completely $(p,1)$-summing (in particular, if u has* cb-*cotype p), then it is $(q',1)$-mixing for any $q > p$. Moreover $\mathfrak{m}^o_{q',1}(u) \leq c(p,q)\pi^{\mathrm{cb}}_{p,1}(u)$, where $c(p,q)$ is a constant depending on p and q only.*

Just from the definition, we obtain a trivial composition formula for completely (q,p)-mixing maps: regardless of the values of p, q and r in $[1,\infty]$, the composition of a completely (p,r)-mixing operator u followed by a completely (q,p)-mixing operator v is completely (q,r)-mixing and moreover $\mathfrak{m}^o_{q,r}(vu) \leq \mathfrak{m}^o_{q,p}(v)\cdot\mathfrak{m}^o_{p,r}(u)$. Many of the properties of completely p-summing maps immediately give rise to corresponding properties of completely (q,p)-mixing maps. For starters, the domination characterization (in its factorization version, as in [Pis98, Rem. 5.7]) for completely p-summing maps implies that for any map u, $\mathfrak{m}^o_{q,p}(u) = \|u\|_{\mathrm{cb}}$ whenever $q \leq p$ and $\mathfrak{m}^o_{\infty,p}(u) = \pi^o_p(u)$, so only the case $1 \leq p < q < \infty$ gives something new. Moreover, completely (q,p)-mixing maps also satisfy the ideal property and $\mathfrak{m}^o_{q,p}(v\circ u\circ w) \leq \|v\|_{\mathrm{cb}}\cdot\mathfrak{m}^o_{q,p}(u)\cdot\|w\|_{\mathrm{cb}}$ whenever the composition makes sense. Additionally, the monotonicity of the completely p-summing norms implies a monotonicity condition for the completely (q,p)-mixing norms: whenever $p_1 \leq p_2$ and $q_2 \leq q_1$, $\mathfrak{m}^o_{q_2,p_2}(u) \leq \mathfrak{m}^o_{q_1,p_1}(u)$ for any u. Finally, being completely (q,p)-mixing is a *local* concept. As in the proof of [Yew08, Prop. 5.(2)], for any map $u : E \to F$,

$$\mathfrak{m}^o_{q,p}(u) = \sup\{\mathfrak{m}^o_{q,p}(u|_{E_0}) \ : \ E_0 \subseteq E, \dim(E_0) < \infty\}.$$

4. Characterizations

4.1. **Domination.** The following theorem is the completely (q,p) mixing counterpart of the domination theorem for completely p-summing maps of Pisier.

Theorem 4.1. *Let $E \subseteq B(H)$ and $F \subseteq B(K)$ be concrete operator spaces. Let $1 \leq p \leq q < \infty$, $u : E \to F$ a linear map and $C \geq 0$. The following are equivalent:*

(a) u is completely (q,p)-mixing with $\mathfrak{m}^o_{q,p}(T) \leq C$.
(b) For any ultrafilter $\mathcal{U}$ over an index set I and families $(a_\alpha)_{\alpha\in I}$, $(b_\alpha)_{\alpha\in I}$ in the unit ball of $S_{2q}(K)$ there exist an index set J, an ultrafilter $\mathcal{V}$ over J and families $(c_\beta)_{\beta\in J}$, $(d_\beta)_{\beta\in J}$ in the unit ball of $S_{2p}(H)$ such that for all n and all (x_{ij}) in $M_n(E)$ we have

$$\lim_{\mathcal{U}} \big\| \big[(a_\alpha(ux_{ij})b_\alpha)\big] \big\|_{M_n(S_q(K))} \leq C \lim_{\mathcal{V}} \big\| \big[(c_\beta x_{ij} d_\beta)\big] \big\|_{M_n(S_p(H))}$$

(c) For any ultrafilter $\mathcal{U}$ over an index set I and families $(a_\alpha)_{\alpha\in I}$, $(b_\alpha)_{\alpha\in I}$ in the unit ball of $S_{2q}(K)$ there exist an index set J, an ultrafilter $\mathcal{V}$ over J and families $(c_\beta)_{\beta\in J}$, $(d_\beta)_{\beta\in J}$ in the unit ball of $S_{2p}(H)$ such that for all

n and all (x_{ij}) in $M_n(E)$ we have

$$\lim_{\mathcal{U}} \left\| \left[(a_\alpha (u x_{ij}) b_\alpha) \right] \right\|_{S_p^n[S_q(K)]} \leq C \lim_{\mathcal{V}} \left\| \left[(c_\beta x_{ij} d_\beta) \right] \right\|_{S_p(\ell_2^n \otimes H)}.$$

Proof. We only show that (a) and (b) are equivalent, the equivalence with (c) follows similarly (as in Pisier's [Pis98] proof of Theorem 2.1).

$(a) \Rightarrow (b)$ Suppose that u is completely (q,p)-mixing, and let I be an index set, $\mathcal{U}$ an ultrafilter over I and $(a_\alpha)_{\alpha \in I}$, $(b_\alpha)_{\alpha \in I}$ families in the unit ball of $S_{2q}(K)$. The ultraproduct m of the multiplication maps $M(a_\alpha, b_\alpha) : B(K) \to S_q(K)$ is completely q-summing with completely q-summing norm at most one and therefore, if j is the completely isometric injection of $B(K)$ into the ultrapower $B(K)^{\mathcal{U}}$, $m \circ j \circ u$ is completely p-summing with $\pi_p^o(m \circ j \circ u) \leq C$. By the domination theorem for completely p-summing maps (Theorem 2.1), there exists an ultrafilter $\mathcal{V}$ over an index set J and families $(c_\beta)_{\beta \in J}$, $(d_\beta)_{\beta \in J}$ in the unit ball of $S_{2p}(H)$ such that for any $n \in \mathbb{N}$ and any (x_{ij}) in $M_n(E)$,

$$\left\| \left[((mju) x_{ij}) \right] \right\|_{M_n[S_q(K)^{\mathcal{U}}]} \leq C \lim_{\mathcal{V}} \left\| \left[(c_\beta x_{ij} d_\beta) \right] \right\|_{M_n(S_p(H))},$$

that is,

$$\lim_{\mathcal{U}} \left\| \left[(a_\alpha (u x_{ij}) b_\alpha) \right] \right\|_{M_n(S_q(K))} \leq C \lim_{\mathcal{V}} \left\| \left[(c_\beta x_{ij} d_\beta) \right] \right\|_{M_n(S_p(H))}.$$

$(b) \Rightarrow (a)$ Let $v : F \to G$ be a completely q-summing map. By the domination theorem for completely q-summing maps, there exists an ultrafilter $\mathcal{U}$ over an index set I and families $(a_\alpha)_{\alpha \in I}$, $(b_\alpha)_{\alpha \in I}$ in the unit ball of $S_{2q}(K)$ such that for any $n \in \mathbb{N}$ and any (y_{ij}) in $M_n(F)$,

$$\left\| \left[(v y_{ij}) \right] \right\|_{M_n[G]} \leq \pi_q^o(v) \lim_{\mathcal{U}} \left\| \left[(a_\alpha y_{ij} b_\alpha) \right] \right\|_{M_n(S_q(K))}.$$

By hypothesis, there exist an index set J, an ultrafilter $\mathcal{V}$ over J and families $(c_\beta)_{\beta \in J}$, $(d_\beta)_{\beta \in J}$ in the unit ball of $S_{2p}(H)$ such that for all n and all (x_{ij}) in $M_n(E)$ we have

$$\lim_{\mathcal{U}} \left\| \left[(a_\alpha (u x_{ij}) b_\alpha) \right] \right\|_{M_n(S_q(K))} \leq C \lim_{\mathcal{V}} \left\| \left[(c_\beta x_{ij} d_\beta) \right] \right\|_{M_n(S_p(H))}.$$

The two previous inequalities put together give us

$$\left\| \left[(v u x_{ij}) \right] \right\|_{M_n[G]} \leq C \pi_q^o(v) \lim_{\mathcal{V}} \left\| \left[(c_\beta x_{ij} d_\beta) \right] \right\|_{M_n(S_p(H))},$$

which means, by the domination theorem for completely p-summing maps, that $v \circ u$ is completely p-summing and $\pi_p^o(v \circ u) \leq C \pi_q^o(v)$, meaning that u is completely (q,p)-mixing with $\mathfrak{m}_{q,p}^o(u) \leq C$.

$\square$

4.2. **Mixed norms.** We will now prove another characterization of completely (q,p)-summing maps, based on mixed S_p-norm inequalities (Theorem 4.3). First we need the following lemma, which is a generalization of [Pis98, Theorem 1.5].

Lemma 4.2. *Suppose $1/p = 1/q + 1/r$. Let $X \in S_p[E]$ (resp. $X \in S_p^n[E]$) and let $(x_{ij}) \in M_\infty(E)$ (resp. $(x_{ij}) \in M_n(E)$) be the corresponding matrix with $x_{ij} \in E$. Then $\|X\|_{S_p[E]}$ (resp. $\|X\|_{S_p^n[E]}$) is equal to*

$$\inf\left\{\|A\|_{S_{2r}}\|V\|_{S_q[E]}\|B\|_{S_{2r}}\right\}$$

where the infimum runs over all representations of the form

$$(x_{ij}) = A\cdot V\cdot B$$

with $A, B \in S_{2r}$ and $V \in S_q[E]$ (resp. $A, B \in S_{2r}^n$ and $V \in M_n(E)$).

Proof. If $(x_{ij}) = A\cdot V\cdot B$, then by [Pis98, Lemma 1.6.(ii)], we have that

$$\|(x_{ij})\|_{S_p[E]} \leq \|A\|_{S_{2r}}\|V\|_{S_q[E]}\|B\|_{S_{2r}},$$

and hence

$$\|(x_{ij})\|_{S_p[E]} \leq \inf\left\{\|A\|_{S_{2r}}\|V\|_{S_q[E]}\|B\|_{S_{2r}}\right\}.$$

For the opposite inequality, recall from [Pis98, Theorem 1.5] that

$$\|(x_{ij})\|_{S_p[E]} = \inf\left\{\|\mathbf{A}\|_{S_{2p}}\|Y\|_{M_\infty[E]}\|\mathbf{B}\|_{S_{2p}} : (x_{ij}) = \mathbf{A}\cdot Y\cdot\mathbf{B}\right\}.$$

Therefore, given $\varepsilon > 0$ there exists a factorization $(x_{ij}) = \mathbf{A}\cdot Y\cdot\mathbf{B}$ such that

$$\|(x_{ij})\|_{S_p[E]} + \varepsilon \geq \|\mathbf{A}\|_{S_{2p}}\|Y\|_{M_\infty[E]}\|\mathbf{B}\|_{S_{2p}}.$$

By [DJT95, Thm. 6.3], we can choose $A', B' \in S_{2q}$ and $A'', B'' \in S_{2r}$ such that $\mathbf{A} = A''\cdot A'$ and $\|\mathbf{A}\|_{S_{2p}}$ is equal to $\|A'\|_{S_{2q}}\|A''\|_{S_{2r}}$, and $\mathbf{B} = B'\cdot B''$ and $\|\mathbf{B}\|_{S_{2p}}$ is equal to $\|B'\|_{S_{2q}}\|B''\|_{S_{2r}}$. Then using [Pis98, Theorem 1.5] again,

$$\begin{aligned}\|\mathbf{A}\|_{S_{2p}}\|Y\|_{M_\infty[E]}\|\mathbf{B}\|_{S_{2p}} &= \|A''\|_{S_{2r}}\|A'\|_{S_{2q}}\|Y\|_{M_\infty[E]}\|B'\|_{S_{2q}}\|B''\|_{S_{2r}}\\ &\geq \|A''\|_{S_{2r}}\|A'\cdot Y\cdot B'\|_{S_q[E]}\|B''\|_{S_{2r}}\\ &\geq \inf\left\{\|A\|_{S_{2r}}\|V\|_{S_q[E]}\|B\|_{S_{2r}} \;:\; (x_{ij}) = A\cdot V\cdot B\right\},\end{aligned}$$

where the last inequality follows from the fact that $A''\cdot A'\cdot Y\cdot B'\cdot B'' = \mathbf{A}\cdot Y\cdot\mathbf{B} = (x_{ij})$. Letting ε go to zero, we get the desired inequality. □

With this lemma we can prove the announced characterization of completely (q,p)-mixing maps, one that has the advantage of not having any ultrafilters involved. As far as we can tell, it does not directly correspond to a known characterization of (q,p)-mixing operators (in the Banach space case).

Theorem 4.3. *Let $E \subseteq B(H)$ and $F \subseteq B(K)$ be concrete operator spaces. Let $1 \le p \le q < \infty$, $u : E \to F$ a linear map and $C \ge 0$. The following are equivalent:*

(a) u is completely (q,p)-mixing with $\mathfrak{m}^o_{q,p}(u) \le C$.
(b) For all n and all (x_{ij}) in $M_n(E)$ we have

$$\sup\left\{ \left\| \big(a(ux_{ij})b\big) \right\|_{S_p[S_q(K)]} \; : \; a,b \in B_{S_{2q}(K)},\ a,b \ge 0 \right\} \le C \left\| (x_{ij}) \right\|_{S_p \otimes_{\min} E}.$$

Proof. $(a) \Rightarrow (b)$ Suppose that u is completely (q,p)-mixing with $\mathfrak{m}^o_{q,p}(u) \le C$. Let a, b be positive elements in the unit ball of $S_{2q}(K)$. By [Pis98, Proposition 5.6], the multiplication map $M(a,b) : B(K) \to S_q(K)$ is completely q-summing with constant at most one, and thus so is its restriction to F. Therefore, the composition $M(a,b) \circ u : E \to S_q(K)$ is completely p-summing with $\pi^o_p(M(a,b) \circ u) \le C$, that is, the norm of the map

$$I_{S_p} \otimes (M(a,b) \circ u) : S_p \otimes_{\min} E \to S_p[S_q(K)]$$

is at most C. This means that for any (x_{ij}) in $M_n(E)$ we have

$$\left\| \big(a(ux_{ij})b\big) \right\|_{S_p[S_q(K)]} \le C \left\| (x_{ij}) \right\|_{S_p \otimes_{\min} E}.$$

Taking the supremum over all a and b we obtain the desired conclusion.

$(b) \Rightarrow (a)$ Suppose that for all n and all (x_{ij}) in $M_n(E)$ we have

$$\sup\left\{ \left\| \big(a(ux_{ij})b\big) \right\|_{S_p[S_q(K)]} \; : \; a,b \in B_{S_{2p}(K)},\ a,b \ge 0 \right\} \le C \left\| (x_{ij}) \right\|_{S_p \otimes_{\min} E}. \tag{4.1}$$

Let $v : F \to G$ be a completely q-summing map. By the domination theorem for completely q-summing maps (Theorem 2.1) and [Pis98, Theorem 1.9], there exist an ultrafilter $\mathcal{U}$ over an index set I and families $(a_\alpha)_{\alpha \in I}$, $(b_\alpha)_{\alpha \in I}$ in the unit ball of $S_{2q}(K)$ such that for all $n \in \mathbb{N}$ and all (y_{ij}) in $M_n(F)$ we have

$$\left\| (vy_{ij}) \right\|_{S^n_q[G]} \le \pi^o_q(v) \lim_{\mathcal{U}} \left\| (a_\alpha y_{ij} b_\alpha) \right\|_{S^n_q[S_q(K)]}. \tag{4.2}$$

In particular, for every (x_{ij}) in $M_n(E)$ we have

$$\left\| (vux_{ij}) \right\|_{S^n_q[G]} \le \pi^o_q(v) \lim_{\mathcal{U}} \left\| \big(a_\alpha (ux_{ij}) b_\alpha\big) \right\|_{S^n_q[S_q(K)]}. \tag{4.3}$$

Let r be such that $1/p = 1/q + 1/r$, and let $\varepsilon > 0$. For each $\alpha \in I$, Lemma 4.2 implies the existence of A_α and B_α positive matrices in the unit sphere of S^n_{2r} such that

$$\left\| A_\alpha \cdot \big(a_\alpha (ux_{ij}) b_\alpha\big) \cdot B_\alpha \right\|_{S^n_q[S_q(K)]} \le (1+\varepsilon) \left\| \big(a_\alpha (ux_{ij}) b_\alpha\big) \right\|_{S^n_p[S_q(K)]}$$

By compactness, the limits $A = \lim_{\mathcal{U}} A_\alpha$ and $B = \lim_{\mathcal{U}} B_\alpha$ exist in the positive part of the unit sphere of S^n_{2r}. It follows then from the previous inequality

that
(4.4)
$$\lim_{\mathcal{U}} \left\| A\cdot \big(a_\alpha(ux_{ij})b_\alpha\big)\cdot B\right\|_{S_q^n[S_q(K)]} \leq (1+\varepsilon)\lim_{\mathcal{U}} \left\|\big(a_\alpha(ux_{ij})b_\alpha\big)\right\|_{S_p^n[S_q(K)]}.$$

Now, using Lemma 4.2 again together with (4.3), (4.4), (4.2) and (4.1) we have

$$\begin{aligned}\|(vux_{ij})\|_{S_p^n[G]} &\leq \|A\cdot(vux_{ij})\cdot B\|_{S_q^n[G]}\\ &\leq \pi_q^o(v)\lim_{\mathcal{U}}\left\|A\cdot\big(a_\alpha(ux_{ij})b_\alpha\big)\cdot B\right\|_{S_q^n[S_q(K)]}\\ &\leq \pi_q^o(v)(1+\varepsilon)\lim_{\mathcal{U}}\left\|\big(a_\alpha(ux_{ij})b_\alpha\big)\right\|_{S_p^n[S_q(K)]}\\ &\leq \pi_q^o(v)(1+\varepsilon)C\,\|(x_{ij})\|_{S_p\otimes_{\min}E}.\end{aligned}$$

Letting ε go to zero, this shows that vu is completely p-summing with $\pi_p^o(vu)\leq C\pi_q^o(v)$. Therefore, $\mathfrak{m}_{q,p}^o(u)\leq C$. □

5. The "interpolation" result

The main result of this section is the following operator space version of [Pie80, Prop. 20.1.13], which will imply an strengthtening of Yew's quantized extrapolation theorem.

Theorem 5.1. *Let $0<\theta<1$ and $1\leq p\leq q_0,q_1<\infty$. Define $1/q := (1-\theta)/q_0+\theta/q_1$. For a map $u: E\to F\subseteq B(K)$,*
$$\mathfrak{m}_{q,p}^o(u)\leq \mathfrak{m}_{q_0,p}^o(u)^{1-\theta}\mathfrak{m}_{q_1,p}^o(u)^{\theta}.$$

Proof. Let $C_0=\mathfrak{m}_{q_0,p}^o(u)$ and $C_1=\mathfrak{m}_{q_1,p}^o(u)$. By Theorem 4.3,
$$\sup\left\{\left\|\big(a(ux_{ij})b\big)\right\|_{S_p[S_{q_0}(K)]} \;:\; a,b\in B_{S_{2q_0}(K)},\ a,b\geq 0\right\}\leq C_0\,\|(x_{ij})\|_{S_p\otimes_{\min}E}.$$
and
$$\sup\left\{\left\|\big(a(ux_{ij})b\big)\right\|_{S_p[S_{q_1}(K)]} \;:\; a,b\in B_{S_{2q_1}(K)},\ a,b\geq 0\right\}\leq C_1\,\|(x_{ij})\|_{S_p\otimes_{\min}E}.$$

Now, by [Yew08, Lemma 7] (or alternatively, as Yew himself says, by a typical application of the Generalized Hadamard three line theorem and the fact that the spaces $S_p(K)$ form an interpolation chain; see [Yew05, Lemma 3.5] for the detailed proof), for any positive a,b in $S_1(K)$ we have
$$\left\|\big(a^{1/2q}(ux_{ij})b^{1/2q}\big)\right\|_{S_p[S_q(K)]}\leq \left\|\big(a^{1/2q_0}(ux_{ij})b^{1/2q_0}\big)\right\|_{S_p[S_{q_0}(K)]}^{1-\theta}\left\|\big(a^{1/2q_1}(ux_{ij})b^{1/2q_1}\big)\right\|_{S_p[S_{q_1}(K)]}^{\theta}.$$

Therefore,

$$\sup\left\{\left\|\big(a(ux_{ij})b\big)\right\|_{S_p[S_q(K)]} \;:\; a,b\in B_{S_{2q}(K)},\ a,b\geq 0\right\}$$
$$\leq \sup\left\{\left\|\big(a(ux_{ij})b\big)\right\|_{S_p[S_{q_0}(K)]} \;:\; a,b\in B_{S_{2q_0}(K)},\ a,b\geq 0\right\}^{1-\theta}$$
$$\cdot \sup\left\{\left\|\big(a(ux_{ij})b\big)\right\|_{S_p[S_{q_1}(K)]} \;:\; a,b\in B_{S_{2q_1}(K)},\ a,b\geq 0\right\}^{\theta}$$

and thus

$$\sup\left\{\left\|\big(a(ux_{ij})b\big)\right\|_{S_p[S_q(K)]} \;:\; a,b\in B_{S_{2q}(K)},\ a,b\geq 0\right\}$$
$$\leq C_0^{1-\theta}C_1^{\theta}\left\|(x_{ij})\right\|_{S_p\otimes_{\min}E}.$$

Another appeal to Theorem 4.3 gives the desired conclusion. □

Let E be an operator space and $1\leq p_0<p_1<q$. In [Yew08, Thm. 8] it is shown that

$$\mathfrak{m}^o_{q,p_0}(id_E)\leq\left[2^{1/p_0}\mathfrak{m}^o_{q,p_1}(id_E)\right]^{1/\theta}.$$

Our next corollary improves on this result by removing the power of 2, while also emphasizing the fact that for identity maps being completely (q,p)-mixing $(q>p)$ is independent of p.

Corollary 5.2. *Let E be an operator space and $1\leq p_0<p_1<q$. Then id_E is (q,p_0)-mixing if and only if it is (q,p_1)-mixing. Moreover,*

$$\mathfrak{m}^o_{q,p_1}(id_E)\leq\mathfrak{m}^o_{q,p_0}(id_E)\leq\mathfrak{m}^o_{q,p_1}(id_E)^{1/\theta},$$

where θ is defined by $1/p_1=(1-\theta)/q+\theta/p_0$.

Proof. The monotonicity property for (q,p)-mixing constants from Section 3 gives $\mathfrak{m}^o_{q,p_1}(id_E)\leq\mathfrak{m}^o_{q,p_0}(id_E)$, whereas the composition property from the same section provides us with the inequality $\mathfrak{m}^o_{q,p_0}(id_E)\leq\mathfrak{m}^o_{q,p_1}(id_E)\cdot\mathfrak{m}^o_{p_1,p_0}(id_E)$. Now, from Theorem 5.1

$$\mathfrak{m}^o_{p_1,p_0}(id_E)\leq\mathfrak{m}^o_{q,p_0}(id_E)^{1-\theta}\cdot\mathfrak{m}^o_{p_0,p_0}(id_E)^{\theta}=\mathfrak{m}^o_{q,p_0}(id_E)^{1-\theta}\cdot 1.$$

So we obtain

$$\mathfrak{m}^o_{q,p_0}(id_E)\leq\mathfrak{m}^o_{q,p_1}(id_E)\cdot\mathfrak{m}^o_{q,p_0}(id_E)^{1-\theta}$$

from where the result follows. □

6. Composition theorems

We now proceed to prove various composition theorems for completely p-summing and completely p-nuclear operators. Our starting point is the following duality due to M. Junge:

Theorem 6.1. [Jun96, Cor. 3.1.3.9] *When E and F are operator spaces and $1 \le p < \infty$, trace duality yields an isometric isomorphism between $\mathcal{N}_p^o(E,F)^*$ and $\Pi_{p'}^o(F, E^{**})$. In the finite-dimensional case, the duality is also true for $p = \infty$.*

From here we can deduce our first composition result, stating that in the finite dimensional setting, the composition of a completely p-summing map and a completely p'-nuclear one is completely 1-nuclear.

Theorem 6.2. *Let $u : E \to F$ and $v : F \to G$ be linear maps between finite-dimensional operator spaces. Then $\nu_1^o(vu) \le \nu_{p'}^o(v)\pi_p^o(u)$ and $\nu_1^o(vu) \le \pi_p^o(v)\nu_{p'}^o(u)$.*

Proof. We only prove the first inequality, the second one may be obtained using an analogous argument. Consider a linear map $w : G \to E$. Then by Theorem 6.1,

$$|\operatorname{tr}(wvu)| \le \pi_p^o(u)\nu_{p'}^o(wv) \le \pi_p^o(u)\nu_{p'}^o(v)\,\|w\|_{\mathrm{cb}}\,.$$

Taking the supremum over all w of cb-norm at most 1, another appeal to Theorem 6.1 (recalling that completely ∞-summing is the same as completely bounded) shows that $\nu_1^o(vu) \le \pi_p^o(u)\nu_{p'}^o(v)$. $\square$

A proof very similar to that of Theorem 6.2, together with the fact that π_2^o is in trace duality with itself [Lee08, Lemma 2.5], allow us to prove the following.

Theorem 6.3. *Let $u : E \to F$ and $v : F \to G$ be completely 2-summing maps. When the operator spaces are finite-dimensional, $\nu_1^o(vu) \le \pi_2^o(v)\pi_2^o(u)$. In the infinite-dimensional case, localization gives $\pi_1^o(vu) \le \pi_2^o(v)\pi_2^o(u)$.*

The following lemma is at the heart of the proof of the composition theorem for completely p-summing operators with conjugate indices.

Lemma 6.4. *Let $u : E \to F \subseteq B(\ell_2)$ be a completely p-summing map and a, b in $S_{2p'}$. Let $M := M(a,b) : B(\ell_2) \to S_{2p'}$ be a multiplication map induced by a and b. Then $\pi_1^o(M \circ u) \le \|a\|_{2p'}\,\|b\|_{2p'}\,\pi_p^o(u)$.*

Proof. Let $\varepsilon > 0$. There exist orthonormal sequences $(e_j), (f_j)$ in ℓ_2 and a sequence of nonnegative numbers (τ_j) such that $|a| = \sum_j \tau_j e_j \otimes f_j$, and $(\sum_j \tau_j^{2p'})^{1/(2p')} = \|a\|_{2p'}$. Let (λ_j) be a sequence of real numbers greater than one and increasing to infinity such that $(\sum_j \lambda_j^{2p'}\tau_j^{2p'})^{1/(2p')} \le (1+\varepsilon)\,\|a\|_{2p'}$. Define $a' = \sum_j \lambda_j \tau_j e_j \otimes f_j$ and let k_1 be the composition of the orthogonal projection onto the span of (e_j) followed by the operator that sends e_j to $\lambda_j^{-1} e_j$. Then we have a decomposition $a = a'k_1$ where k_1 is compact with $\|k_1\| \le 1$ and $\|a'\|_{2p'} \le (1+\varepsilon)\,\|a\|_{2p'}$. Similarly, we can find a decomposition $b = k_2 b'$ where k_2 is compact with norm at most 1 and $\|b'\|_{2p'} \le (1+\varepsilon)\,\|b\|_{2p'}$.

Therefore, we may factor $M \circ u = M' \circ M'' \circ u$, where $M'' := M(k_1, k_2) : B(\ell_2) \to S_\infty$ and $M' := M(a', b') : S_\infty \to S_{2p'}$. Note that M' is completely p'-nuclear, and $\nu^o_{p'}(M') \leq \|a'\|_{2p'} \|b'\|_{2p'}$. From [Oik10], $\|M''\|_{\mathrm{cb}} \leq \|k_1\| \, \|k_2\|$. By localization we may assume that E is finite-dimensional, and thus by the proof of Theorem 6.2, $\nu^o_1(M' \circ M'' \circ u) \leq \nu^o_{p'}(M')\pi^o_p(M'' \circ u)$. Since $\pi^o_1(M \circ u) \leq \nu^o_1(M' \circ M'' \circ u)$, we have

$$\begin{aligned}\pi^o_1(M \circ u) &\leq \|a'\|_{2p'} \|b'\|_{2p'} \, \|M''\|_{\mathrm{cb}} \, \pi^o_p(u) \\ &\leq \|a'\|_{2p'} \|b'\|_{2p'} \, \|k_1\| \, \|k_2\| \, \pi^o_p(u) \leq (1+\varepsilon)^2 \, \|a\|_{2p'} \, \|b\|_{2p'} \, \pi^o_p(u).\end{aligned}$$

Letting ε go to 0, we get the desired result. □

Now we can prove the composition theorem for completely p-summing operators in the case of conjugate indices.

Theorem 6.5. *Let $u : E \to F$ be completely p-summing and $v : F \to G$ be completely p'-summing. Then vu is completely 1-summing, and moreover $\pi^o_1(vu) \leq \pi^o_{p'}(v)\pi^o_p(u)$.*

Proof. By localization, we can assume that the operator spaces are finite-dimensional and thus $F \subset B(\ell_2)$. Hence, the result follows immediately from Theorem 4.3 and Lemma 6.4. □

We will obtain the full composition theorem from the particular case of conjugate indices using interpolation. Before proceeding to the argument, let us recall [Pis03, Corollary 2.7.7], which states that

$$(X \otimes_{\min} E_0, X \otimes_{\min} E_1)_\theta = X \otimes_{\min} (E_0, E_1)_\theta$$

whenever X is a completely complemented subspace of S_∞.

Lemma 6.6. *Let $1 \leq p, q, r \leq \infty$ with $1/r = 1/p + 1/q$. For a completely p-summing map $u : S_\infty \to F$ and any completely q-summing map $v : F \to G$ we have $\pi^o_r(vu) \leq \pi^o_q(v)\pi^o_p(u)$.*

Proof. If $r = 1$ the result follows from Theorem 6.5, so we may assume $r > 1$. Note that then $p' < q$, so $\theta = p'/q$ is in $(0, 1)$. Consider a completely isometric embedding $J : F \to B(K)$. Define a multilinear map $\Phi : (S^n_t \otimes_{\min} S_\infty) \times S_{2s}(K) \times S_{2s}(K) \to S^n_t[S_s(K)]$ by

$$\Phi\big((x_{ij}), a, b\big) = \big(a(Jux_{ij})b\big).$$

By Theorem 6.5 we have

$$\big\|\Phi\big((x_{ij}), a, b\big)\big\|_{S^n_1[S_{p'}(K)]} \leq \|(x_{ij})\|_{S^n_1 \otimes_{\min} S_\infty} \, \|a\|_{S_{2p'}(K)} \, \|b\|_{S_{2p'}(K)}$$

for any $(x_{ij}) \in S^n_1 \otimes S_\infty$ and $a, b \in S_{2p'}(K)$, that is, Φ has norm at most 1 when $t = 1$ and $s = p'$. Similarly, by the ideal property for completely p-summing operators Φ has norm at most 1 when $t = p$, $s = \infty$. Observe that $1/q = (1-\theta)/\infty + \theta/p'$ and $1/r = (1-\theta)/p + \theta/1$. Therefore, multilinear complex interpolation gives that Φ has norm ≤ 1 when $t=r$ and $s=q$. From Theorem 4.3, we obtain that u has completely (q,r)-mixing norm at most $\pi_p^o(u)$, the desired result. $\square$

Let us now apply the previous lemma to estimate the completely (q,r)-mixing norm of completely p-nuclear operators.

Lemma 6.7. *Let $1 \leq p,q,r \leq \infty$ with $1/r = 1/p + 1/q$, and $u : E \to F$ be a completely p-nuclear map. Then $\mathfrak{m}^o_{q,r}(u) \leq \nu^o_p(u)$.*

Proof. Consider a completely p-nuclear factorization of $u : E \to F$ as

$$E \xrightarrow{\alpha} S_\infty \xrightarrow{M(a,b)} S_p \xrightarrow{\beta} F$$

with $a,b \in S_{2p}$, and let $v : F \to G$ be a completely q-summing map. By Lemma 6.6,

$$\pi_r^o\big(v\beta M(a,b)\big) \leq \pi_q^o(v)\pi_p^o\big(\beta M(a,b)\big) \leq \pi_q^o(v)\left\|\beta\right\|_{\mathrm{cb}}\left\|a\right\|_{S_{2p}}\left\|b\right\|_{S_{2p}}.$$

Thus, by the ideal property for completely p-summing operators

$$\pi_r^o(vu) \leq \left\|\alpha\right\|_{\mathrm{cb}}\pi_r^o\big(v\beta M(a,b)\big) \leq \pi_q^o(v)\left\|\alpha\right\|_{\mathrm{cb}}\left\|\beta\right\|_{\mathrm{cb}}\left\|a\right\|_{S_{2p}}\left\|b\right\|_{S_{2p}}.$$

Taking the infimum over all such representations of u we obtain $\pi_r^o(vu) \leq \pi_q^o(v)\nu_p^o(u)$ giving the desired result. $\square$

Together with the duality theorem, the previous lemmas will yield the full composition theorem.

Theorem 6.8. *Let $1 \leq p,q,r, \leq \infty$ with $1/r = 1/p + 1/q$. Let $u : E \to F$ be completely p-summing and $v : F \to G$ be completely q-summing. Then vu is completely r-summing, and moreover $\pi_r^o(vu) \leq \pi_q^o(v)\pi_p^o(u)$.*

Proof. By localization, we may assume that all the operator spaces involved are finite-dimensional, so in particular we can assume $F \subseteq B(\ell_2)$. By Theorem 4.3 we may assume that v is of the form $M(a,b) : B(\ell_2) \to S_q$ where a and b are in the unit ball of S_{2q}, and thus $\nu_q^o\big(M(a,b)\big) \leq 1$. Let $w : S_q \to E$ be completely r'-nuclear with $\nu_{r'}^o(w) \leq 1$. By Theorem 6.1,

$$|\operatorname{tr}(vuw)| \leq \nu_q^o(v)\pi_{q'}^o(uw).$$

Since $1/q' = 1/p + 1/r'$, Lemma 6.7 implies that

$$|\operatorname{tr}(vuw)| \leq \nu_q^o(v)\pi_p^o(u)\nu_{r'}^o(w) \leq \pi_p^o(u).$$

Taking the supremum over all w with $\nu_{r'}^o(w) \leq 1$, the duality theorem 6.1 gives $\pi_r^o(vu) \leq \pi_p^o(u)$, and the result follows. $\square$

As an application, we now prove an operator space version of [DF93, 32.2.(3)], which in turn is part of a result of Saphar [Sap72].

Corollary 6.9. *For an operator space E and $1 \leq q \leq \infty$, id_E is completely $(q,1)$-mixing if and only if $CB(S_\infty, E) = \Pi_{q'}^o(S_\infty, E)$.*

Proof. First, suppose that id_E is completely $(q,1)$-mixing. By localization, it suffices to prove that there is a constant C such that for all n and all $w : M_n \to E$ we have $\pi^o_{q'}(w) \leq C \|w\|_{\mathrm{cb}}$.

We need to show that w is completely q'-summing, so we might as well assume that E is finite-dimensional. Let $v : E \to M_n$ be a completely q-nuclear map (hence completely q-summing). Since E is completely $(q,1)$-mixing, v is completely 1-summing and moreover $\pi^o_1(v) \leq \nu^o_q(v)\mathfrak{m}^o_{q,1}$. Applying the duality theorem 6.1 for two different pairs of conjugate indices (q and q', 1 and ∞) we have

$$\begin{aligned}\pi^o_{q'}(w) \leq \nu^o_{q'}(w) = \sup\big\{|\operatorname{tr}(vw)| \ : \ \pi^o_q(v : E \to M_n) \leq 1\big\}\\ \leq \mathfrak{m}^o_{q,1}(E)\sup\big\{|\operatorname{tr}(vw)| \ : \ \pi^o_1(v : E \to M_n) \leq 1\big\}\\ = \mathfrak{m}^o_{q,1}(E)\nu^o_\infty(w) \leq \mathfrak{m}^o_{q,1}(E)\|w\|_{\mathrm{cb}},\end{aligned}$$

where in the last step we have used that $\nu^o_\infty(w) = \|w\|_{\mathrm{cb}}$, obvious since w has domain M_n.

Now suppose that $CB(S_\infty, E) = \Pi^o_{q'}(S_\infty, E)$. By the closed graph theorem, there exists a constant C such that for all $w : S_\infty \to E$ we have $\pi^o_{q'}(w) \leq C\|w\|_{\mathrm{cb}}$. Let $v : E \to F$ be a completely q-summing map. Let $n \in \mathbb{N}$ and $w : M_n \to E$ be a completely bounded map. By the assumption, $\pi^o_{q'}(w) \leq C\|w\|_{\mathrm{cb}}$. By the composition theorem 6.8, $\pi^o_1(vw) \leq \pi^o_q(v)\pi^o_{q'}(w) \leq \pi^o_q(v)C\|w\|_{\mathrm{cb}}$. Taking the supremum over all n and all such maps w with cb-norm at most one, we find that $\pi^o_1(v) \leq C\pi^o_q(v)$. Therefore, id_E is completely $(q,1)$-mixing with constant at most C. ☐

We finish the paper with a natural open question. In the Banach space setting, there are other composition formulas for p-summing, p-nuclear and p-integral maps (see [PP69]). Do their operator space analogues hold? Specifically, do we have $\nu^o_r(vu) \leq \pi^o_q(v)\nu^o_p(u)$, $\nu^o_r(vu) \leq \nu^o_q(v)\pi^o_p(u)$, $\iota^o_r(vu) \leq \pi^o_q(v)\iota^o_p(u)$, $\iota^o_r(vu) \leq \iota^o_q(v)\pi^o_p(u)$ whenever the compositions make sense?

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Department of Mathematics, Texas A&M University, College Station, Texas, 77843.

E-mail address: `jcdom@math.tamu.edu`